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TENSORIAL CALCULATION APPLICATION IN ANINSURANCE PROBLEM: AN EXAMPLE OF MULTIDIMENSIONAL CORRELATIONS AND RISK AGGREGATION ANALYSIS

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ABSTRACT

Tensor methods are not commonly used for standard insurance pricing or reserve calculations, which often rely on traditional statistical and actuarial techniques. However, they can be highly useful in advanced scenarios with multidimensional data structures, nonlinear correlations, or complex interactions among different data types.

KEYWORDS:- Tensor calculations, multidimensional data, comprehensive actuarial risk interactions, matrix processing.

I. GENERAL CONTEXT:

We assume an insurance company provides multiple lines of coverage (life, cars, health, property, etc.). Each line has its own loss distribution, and correlations can exist among them. For example, an economic downturn might simultaneously increase health related expenses and auto accident frequency.

Typically, these correlations are modeled using correlation or covariance matrices second order objects. However, if the relationship among variables is not purely linear or includes higher order effects, higher rank tensors are needed to capture these complexities.

- A covariance matrix is a second order tensor that describes how a set of random variables varies jointly.

- To handle more **intricate dependencies**, such as **distributions distorted by extreme events** across several lines, one may employ third of fourth order tensors.

Although classic statistical and actuarial approaches (GLMs, parametric distributions, Monte Carlo simulations, etc.) dominate current practice, tensor calculations provide a robust theoretical framework to address the increasing complexity in risk aggregation:

- They enable a more comprehensive analysis of interdependencies.
- They facilitate the modeling of extreme events, especially when multiple lines can experience simultaneous or interdependent severe losses.
- They provide a geometric lens through which to assess the sensitivity of risk models particularly relevant in setting with high uncertainty and financial volatility.

Thus, in an insurance problem involving multiple correlated lines that requires a thorough understanding of risk accumulation and capital determination (whether for internal economic capital or regulatory requirements), tensor-based techniques offer a broader mathematical perspective on interactions and overall loss aggregation.

II. BASIC MODEL: AGREGATION OF RIESGOS AND ECONOMICAL CAPITAL

1. Defining the Loss vector:

1.1 We define a vector $X = (X_1, X_2, \dots, X_n)$which $X_i$is the random loss (total claims cost) in line over a given period (for example, one year).

2. Second order correlation/covariance tensor:

2.1 Traditional analysis uses the covariance matrix $\Sigma_{ij} = Cov(X_i, X_j)$, which can be viewed as a second order tensor.

2.2 Tensor calculations allows $\Sigma\backslash sigma\Sigma$to be manipulated for purposes such as determining how the variance of an aggregate of lines is decomposed.

3. Extension to higher order tensions:

3.1 When dependencies among lines are not purely linear, if there are heavy tail effects or asymmetric, non-Gaussian dependencies, higher order cumulants (third or fourth order) can be introduced to capture features like fat tails, skewness and joint probability spikes.

3.2 Each cumulant of order kkk is represented by a rank kkk tensor describing the multilinear relationships among kkk random variables.

4. Risk Measures and Economic Capital:

4.1 In insurance, typical risk measures include VaR (Value at Risk) and TvaR (Tail Value at Risk). Determining the required economic capital, funds set aside to cover extremely high losses across different lines combine.

4.2 A tensor based perspective helps us see how small changes in the correlation structure (or in the tensors of higher order cumulants) impact total risk aggregation and consequently, the capital reserve needed.

5. Geometric interpretation

5.1 From the standpoint of tensor calculus and differential geometry, the joint distribution can be thought of as a "space" with certain statistical curvature. This curvature indicates how sensitive model´s conclusions (capital estimates) are to parameter shifts.

5.2 A metric tensor (or statistical metric) can be employed to describe how risk measures respond to tiny changes in the joint distribution.

III. TENSORIAL MODELING. EXAMPLE OF ILLUSTRATIONS (2 LINES)

Assume:

1. $X = (X_1, X_2)$It is the vector of annual losses of your safety LINES.
2. Each $X_i$has:
   - Media$E[X_i]$
   - Variance VaR $VaR(X_i)$
   - Third central moment $\mu_3(X_i) = E[(X_i - E[X_i])^3]$indicating skewness.
3. X_1There is a correlation of second order (covariance) between them.X_2

Numerical values:

For X_1:

- $E[X_1] = 100$
- $Var\ (X_1) = 25$

(Standard deviation $\sigma X_1 = 5$)

- Third central moment$\mu_3(X_1) = 125$
- Note: If you are concerned about the a dimensional X_1,skewness of the series:

$$skew(X_1) = \boxed{\frac{\mu_3(X_1)}{\sigma_{X_1}^3} = \frac{125}{5^3} = \frac{125}{125} = 1} \quad (1)$$

For X_2:

- $E[X_2] = 50$
- $Var(X_2) = 9$

(Standard deviation$\sigma X_2 = 3$)

- Third central moment$\mu_3(X_2) = 54$
- $skew\ (X_2) = \frac{54}{3^3} = \frac{54}{27} = 2$

4. Order tensor 2 (covariance matrix)

This is the classic object of the second order:

$$\boxed{\Sigma_{ij} = \begin{pmatrix} \mathrm{Var}(X_1) & \mathrm{Cov}(X_1, X_2) \\ \mathrm{Cov}(X_1, X_2) & \mathrm{Var}(X_2) \end{pmatrix} = \begin{pmatrix} 25 & 9 \\ 9 & 9 \end{pmatrix}} \quad (2)$$

Rank 2 tensor.Σ_{ij}

5. Tensor of order 3 (associated with the terceros cumulantes).

- $K_{1,1,1} = \mu_3(X_1) = 125$
- $K_{2,2,2} = \mu_3(X_2) = 54$
- All the others$K_{i,j,k} = 0$

In an esquematic way, we can think of it $k$as a cube $2 \times 2 \times 2$, but it has unique values and no one has anything to do with $K_{1,1,1}$it.$K_{2,2,2}$

6. Aggregate Risk: $S = X_1 + X_2$

As a safety issue, we are interested in the total loss S. From the tensorial perspective:

- **Media ofSSS**

$$\boxed{E[S] = E[X_1 + X_2] = E[X_1] + E[X_2] = 100 + 50 = 150} \quad (3)$$

- **Variant ofSSS**

$$\boxed{Var(S) = Var\ (X_1) + Var(X_2) + 2Cov(X_1, X_2)} \quad (4)$$

- **Third central moment ofSSS**

When the cruxes are null, the formula is simplified to $S = X_1 + X_2$be the sum of the three central moments of each variable (most possible mixed endings, for example its 0).

$$\mu_3(S) = \mu_3(X_1) + \mu_3(X_2) + 3_{k_{1,1,2}} + 3_{k_{1,2,2}} = 125 + 54 = 179 \quad (5)$$

IV. DEFINITION OF THE PARAMETERS OF CADA RAMO (7 LINES)

Here's a more complete example with 7 lines.

Definitions of our safety devices and their statistical characteristics:

- **Expected Losses:**$(\boldsymbol{E}[\boldsymbol{X_i}])$:

$$E[X] = (500,300,400,250,350,200,450)$$

- **Variation of each Line**$\left(\boldsymbol{Var}\ (\boldsymbol{X_i})\right)$:

$$\sigma^2 = (100^2, 80^2, 90^2, 70^2, 85^2, 60^2, 95^2) \quad (6)$$

(Standard routes correspond to their$\sigma_i = (100,80,90,70,85,60,95)$

- **Correlations Among the 7 Lines** $\left(\rho_{i,j}\right)$

  To simplify, let's summarize the following matrix of correlations between the 7 rows:

$$\rho = \begin{bmatrix} 1.00 & 0.3 & 0.2 & 0.4 & 0.1 & 0.25 & 0.15 \\ 0.3 & 1.00 & 0.35 & 0.2 & 0.4 & 0.15 & 0.3 \\ 0.2 & 0.35 & 1.00 & 0.1 & 0.25 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.1 & 1.00 & 0.3 & 0.25 & 0.4 \\ 0.1 & 0.4 & 0.25 & 0.3 & 1.00 & 0.2 & 0.3 \\ 0.25 & 0.15 & 0.3 & 0.25 & 0.2 & 1.00 & 0.2 \\ 0.15 & 0.3 & 0.2 & 0.4 & 0.3 & 0.2 & 1.00 \end{bmatrix}$$

- **Covariance Matrix $\boldsymbol{\Sigma_{i,j}}$(:)**

To simplify, let's summarize the following matrix of correlations between the 7 rows:
Using the relationship:

$$\boxed{\Sigma_{i,j} = \rho_{i,j} * \sigma_i * \sigma_j} \quad (7)$$

We can clearly observe the impact of the connections between each of us.
Definimos the sum of losses of the 7 rows:

$$\boxed{S = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7} \quad (8)$$

The moments of Scalculation are as follows:

1. Media of $\boldsymbol{SSS}$:

$$\boldsymbol{E}[S] = \sum E[X_i] = 500 + 300 + 400 + 250 + 350 + 200 + 450 = 2450$$

2. Varianza of $\boldsymbol{S}$ $(\boldsymbol{usando} \sum \boldsymbol{ij}$:

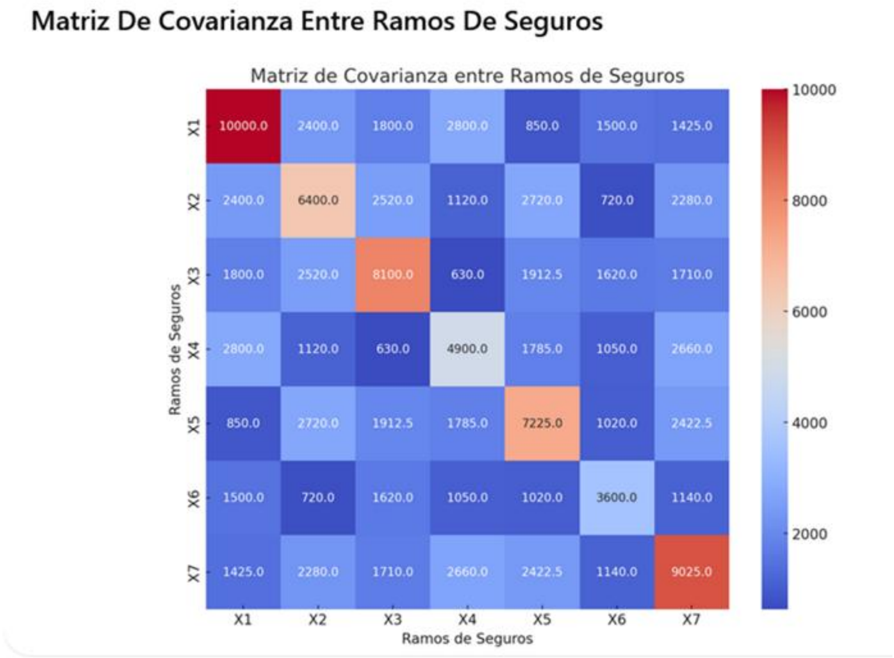
$$\boxed{Var(S) = \sum \Sigma_{ij}\ X_7} \quad (9)$$

To sum up the elements of $\boldsymbol{\Sigma}$, we obtain:

$$Var(S) = 50252{,}5$$

Why:

$$\sigma_s \approx 224{,}2$$



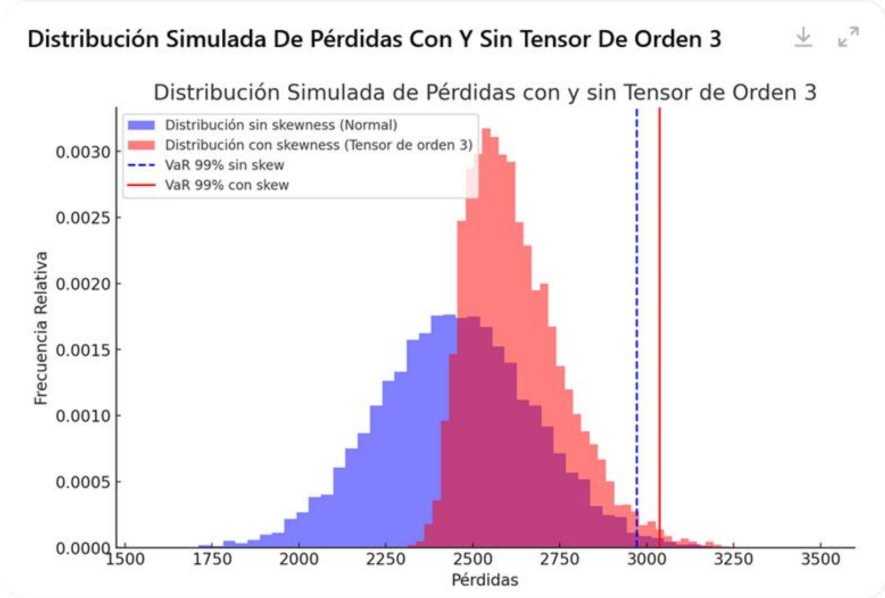
MONTECARLO SIMULATION:

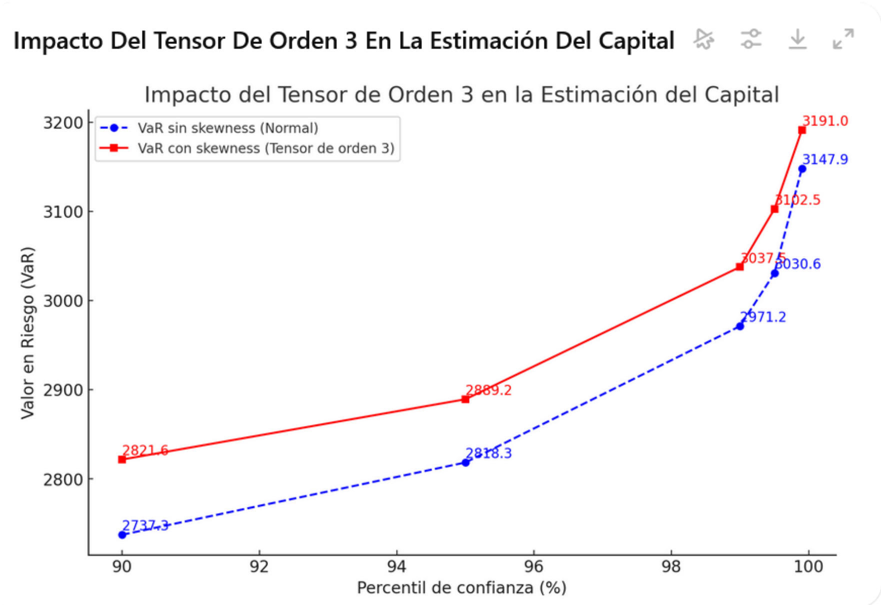
A Monte Carlo simulation illustrates the distribution:

- **Blue dashed curve:** Depicts the normal assumption (symmetric, ignoring skewness).
- **Solid red curve:** Incorporates the third order tensor to account for skewness.
- **Red shaded region:** Demonstrates the risk understated by a purely normal model.

VaR Estimation

- **Blue dashed curve:** VaR under normal assumptions (mean and variance only).
- **Solid red curve:** VaR adjusted for third order effects (skewness).
- **Observations at extreme confidence levels (99%-99.9%).** The adjusted VaR (red) is notably higher, capturing more extreme loss potential.





We will then analyze the tensor time series for capital estimation and observe:

The discontinuous blue curve: Represents the **VaR estimate without skewness** (Using only the media and the variation, you decide, a normal distribution.

The solid red curve: Represents the **VaR corregido with the tensor of order 3,** considering the asimetry.

The difference visible between ambos enfoques:

- For lower confidence levels (90%-95%), there is no difference.

- Please note that our measurements are at extreme levels (99%-99.9%), the **adjusted VaR is significantly higher,** reflecting an **extreme risk** higher when it is incorporated into the **tensor of order 3.**

Notes with specific values:

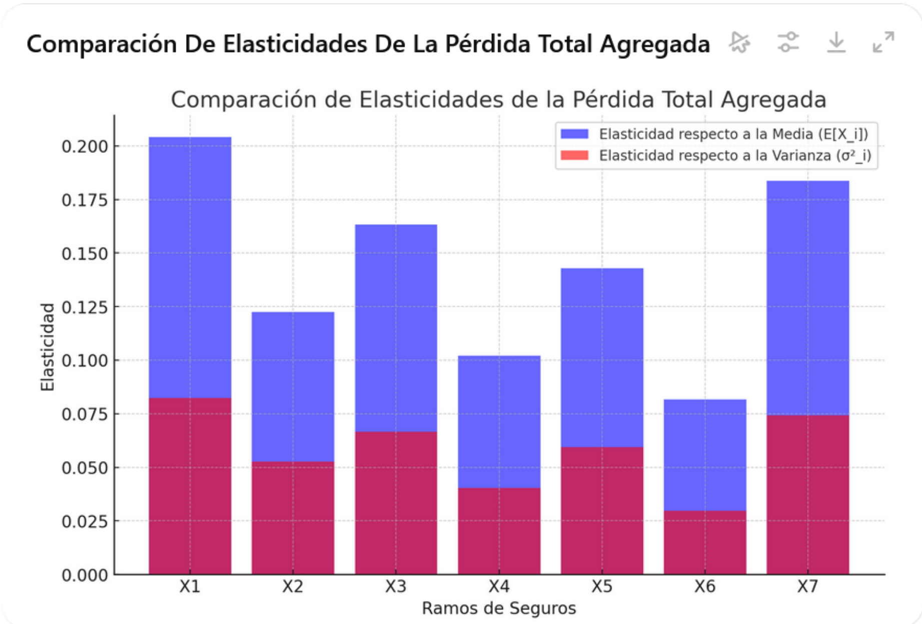
- Show the exact values of the VaR estimated in each percentile to facilitate comparison.

To determine the elasticity of the total loss aggregated $S$with respect to the different risk factors, we analyze the sensitivity of the Sante changes into the variables of the model:

- Elasticity with respect to the media$E[X_i]$
- Elasticity with respect to variationσ_i^2
- Elasticity with respect to correlationρ_{ij}

Elasticity is defined as the proportion of the current change in the dependent variable (aggregation S) with respect to the current change in the explanatory variable.

$$\boxed{E_{X_i} = \frac{\partial S}{\partial X_i} \times \frac{X_i}{S}} \quad (10)$$



Interpretation of the elasticity graphic

The graphic compares the elasticity of the total loss aggregated $S$with respect to the media $E[X_i]$and the variation $\sigma_i^2$of each safety ram.

1. Elasticity with respect to the media (blue bars):

- Indicates how long the total loss aggregated before a current change in the media of each ramo.
- Ramos with greater elasticity in this category are a more significant contribution to the expected value of the total loss.

- Example: If a ram has an elasticity of 0.2 means that an increase of 1% in its media will increase the total loss by 0.2%.

2. Initial weight of each branch

$$\sum_i^{(0)} = \frac{E[X_i]}{\sum_{j=1}^{7} E[X_j]} \quad (11)$$

3. Adjust your weight based on the elasticity of the variation

To minimize the total cost, we adjust the weights to reduce the weight share with greater impact on the total variation:

$$w_i^{(ajustado)} = \frac{w_i^{(0)}}{1 + Elasticidad\sigma_i^2} \quad (12)$$

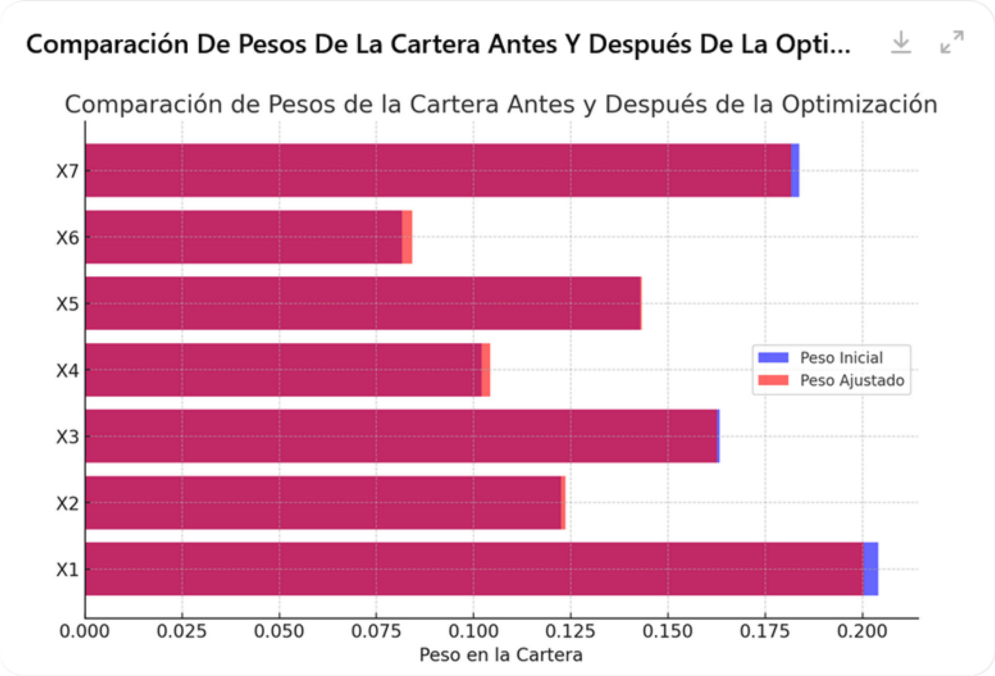
Where:

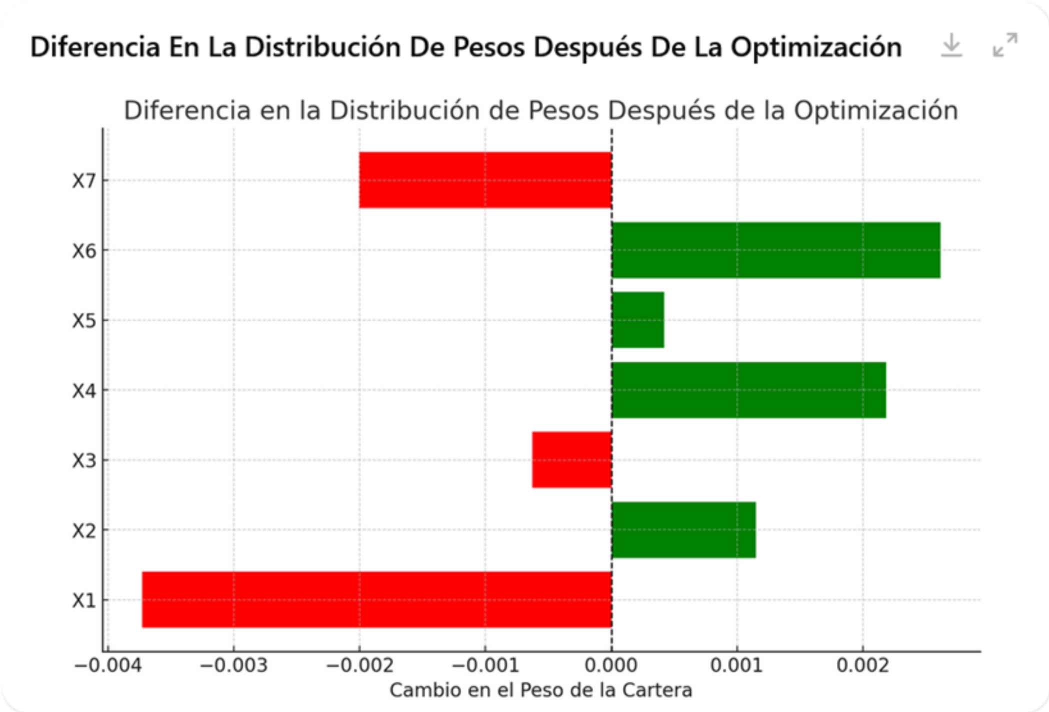
- $w_i^{(ajustado)}$ It's the new weight of the ram i in the cartera.
- $Elasticidad\sigma_i^2$ is the elasticity of the ram with respect to its variation σ_i^2

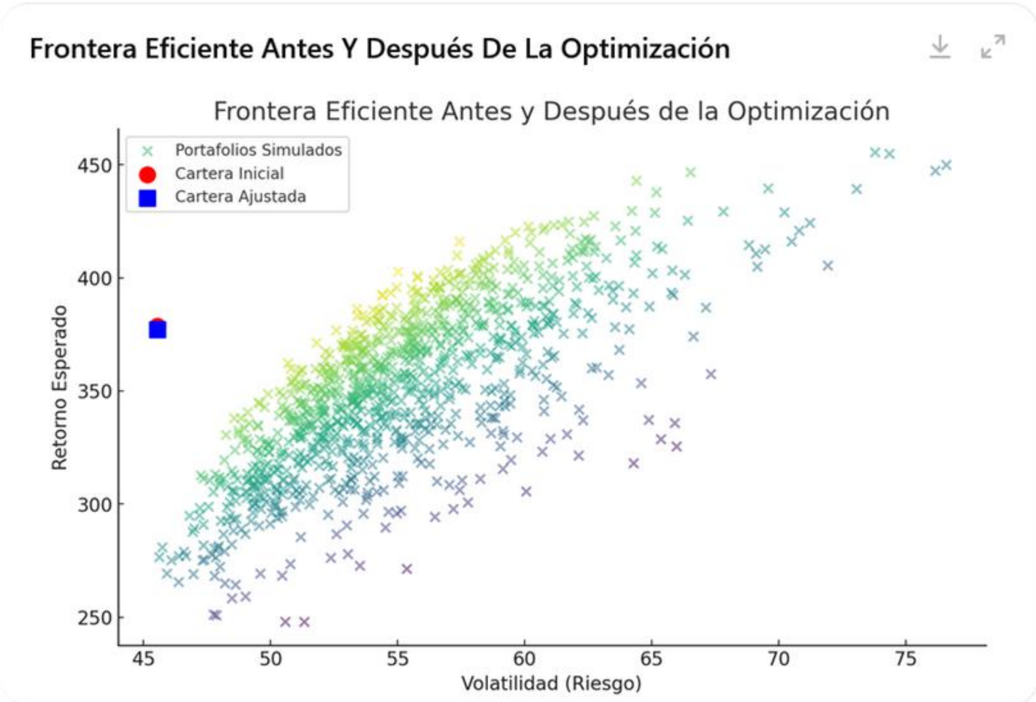
4. Normalization of Adjusted Pesos

So the sum of the pesos should be 1, we normalize the values obtained:

$$w_i^{(final)} = \frac{w_i^{(ajustado)}}{\sum_{j=1}^{7} w_j^{(ajustado)}} \quad (13)$$







- **Scatter Points:** Represents portfolios simulated with distinct combinations of weights between banks.
- **Red circle:** Indicates the position of the initial cover before the adjustments, most of its expected return and volatility.
- **Blue circle:** Represents the optimized card, which has been adjusted to minimize the risk without eliminating any damage.
- **X-axis (volatility):** Mide el riesgo total del portafolio.
- **Y-axis (Expected Return):** Represents the expected profitability of each portfolio.

V. CONCLUSIONS

1. The use of the third-order tensor increases the estimated VaR, showing that, in the presence of positive skewness, the insurer should set aside more capital to cover potential extreme losses that would be underestimated under a second-order (variance-only) approach.
2. If the insurer ignores the asymmetry in risk aggregation, it will underestimate the probability of extreme loss events.
3. Including third-order cumulants allows for a better estimate of the capital needed to cover adverse scenarios.
4. The adjusted VaR is higher, indicating that the insurer should set aside more capital than an estimate based only on variance would suggest.
5. If the insurer uses only the normal approach, it underestimates the probability of catastrophic events.
6. The third-order tensor model predicts fatter tails, which impacts:
    a. Better economic capital planning.
    b. A more robust reinsurance policy.
    c. Greater coverage for extreme loss scenarios.
7. Lines with high covariance can amplify aggregate risk, increasing the capital reserve needed to cover potential losses.
8. Lines with low covariance can offer diversification opportunities, allowing for optimized risk management and reinsurance purchasing.
9. This analysis is key in risk aggregation and solvency models for insurers, as it allows for assessing how much capital should be reserved to avoid financial problems in adverse scenarios.

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